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Evaluating Posterior Probabilities of Mental Models

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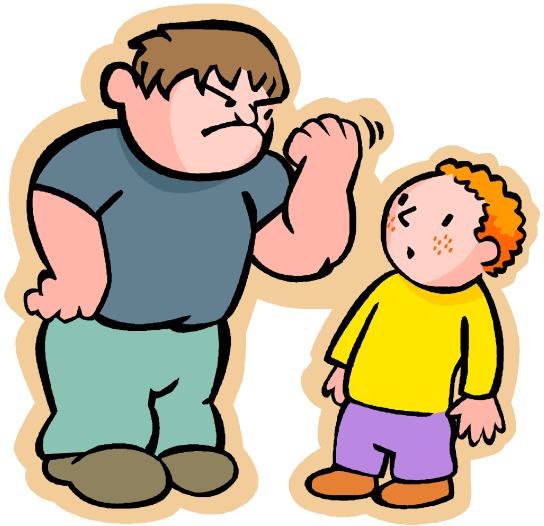
Schoolyard Scenario

Onlooker

Teacher

Bully



Victim

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Bully's Thought Process

Bully observes teacher punishing class

Bully updates estimation of teacher to very strict

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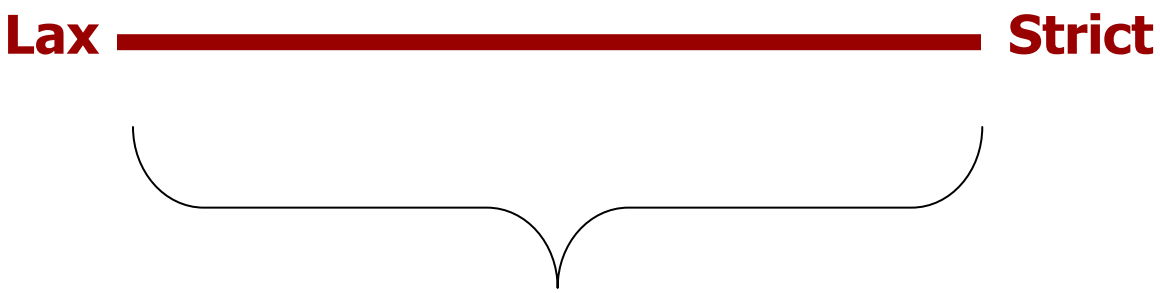
Teacher's Mental Model Space



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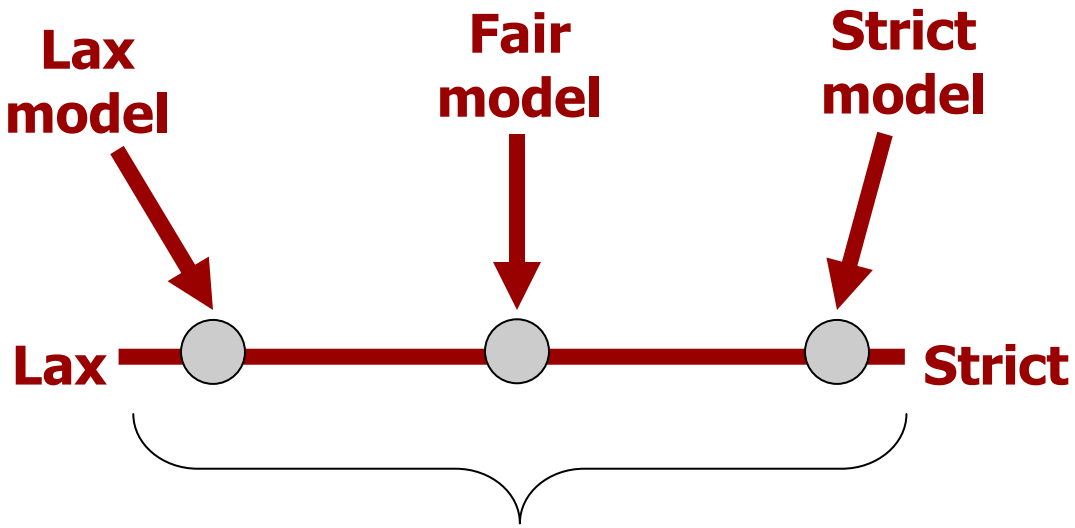
What Does the Bully Consider?

- **Continuous space of mental models is too big!**
  - Must choose a discrete number of mental models to partition the space



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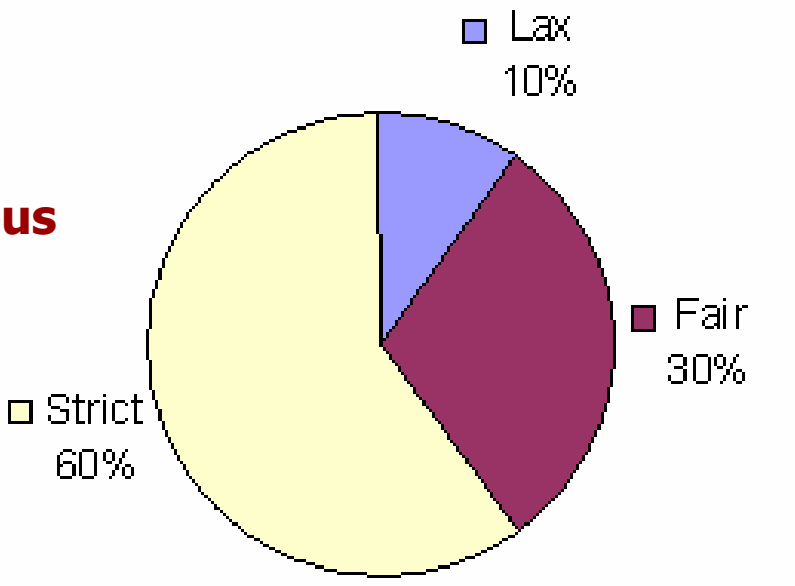
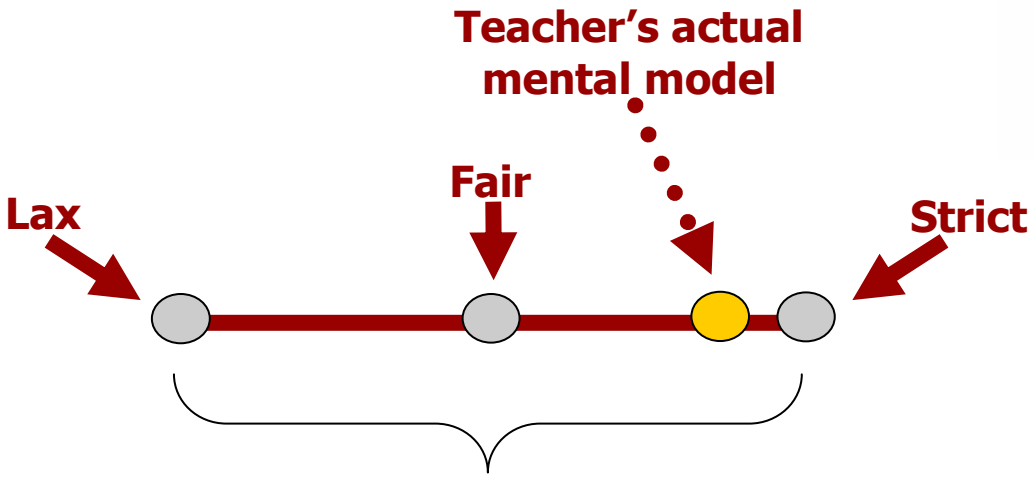
What Does the Bully Believe?

- Choosing 1 mental model is too coarse
- Use a distribution instead!
- Can't have a distribution over continuous space





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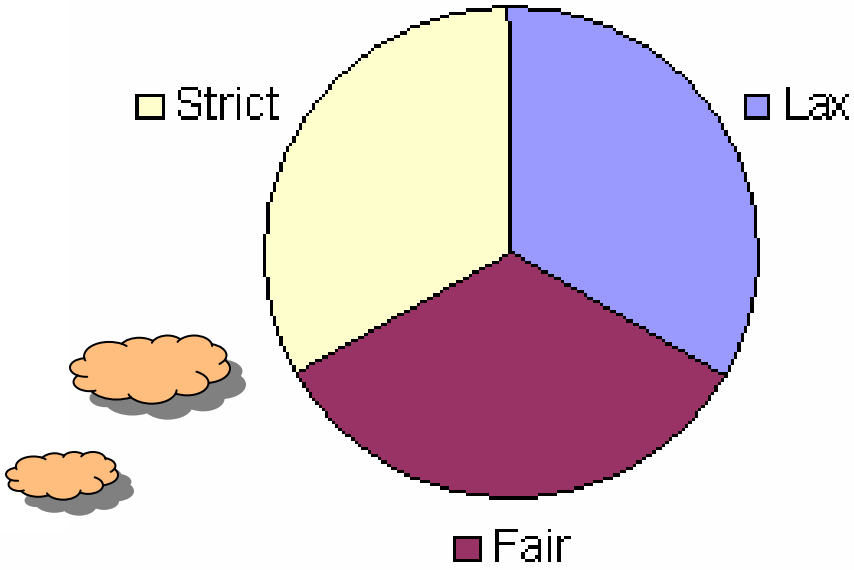
Example – Initial Beliefs

- **Bully has some initial estimation of teacher's mental models**



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Example – Actions and Observations

- Bully takes and observes actions in the world

Bully picks on Victim

Onlooker laughs at Victim

Teacher punishes bully

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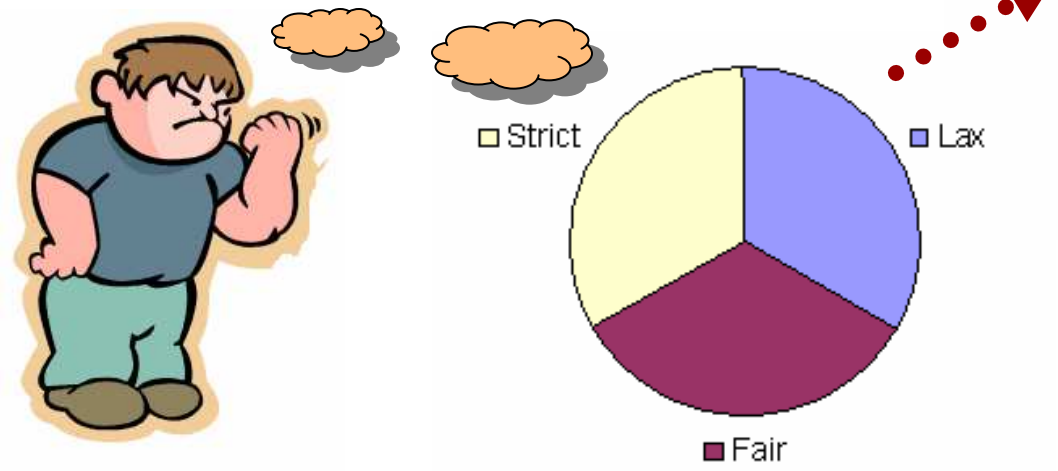
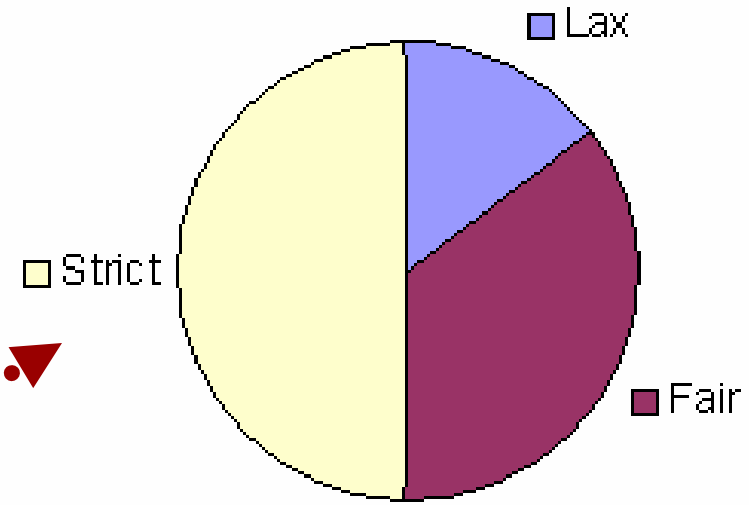
Example: Updating Distribution

- **Based on his punishment, bully updates his probability distribution over teacher's mental models**





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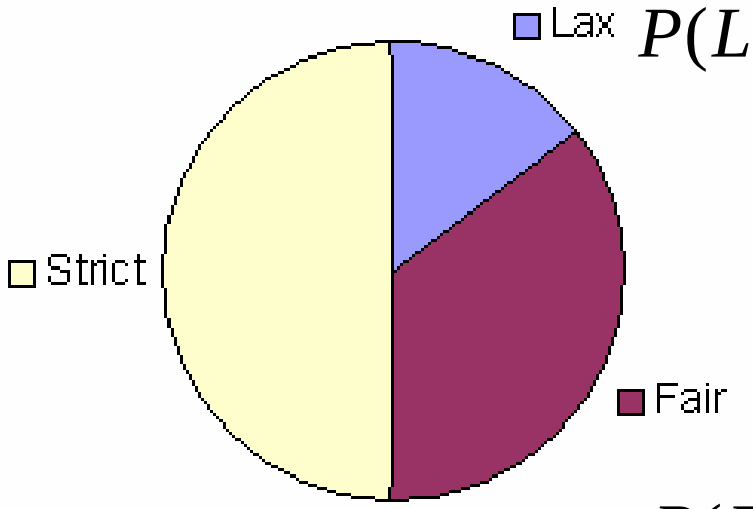
Posterior Probabilities

$P(StrictTeacher \mid PunishBully)$



$P(LaxTeacher \mid PunishBully)$

$P(FairTeacher \mid PunishBully)$

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Calculating Posterior Probabilities

$$P(StrictTeacher \mid PunishBully) = \frac{P(StrictTeacher) \times P(PunishBully \mid StrictTeacher)}{\sum_i P(mentalModel_i) \times P(PunishBully \mid mentalModel_i)}$$

Prior Belief

Conditional Probability

?

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Calculating Conditional Probability

- Conditional probability data not directly available
- However, bully can calculate teacher's expected values for a given action under different mental models

Table of Expected Values

Action	Lax	Fair	Strict
Punish Bully	.5	.75	.75
Punish Class	.4	.6	.5
Punish Observer	.3	.4	.6
Do Nothing	.8	.25	.3

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Expected Value to Conditional Probability

Bully observes teacher punishing him

$$P(PunishBully \mid StrictTeacher)$$

?

Table of Expected Values

Action	Lax	Fair	Strict
Punish Bully	.5	.75	.75
Punish Class	.4	.6	.5
Punish Observer	.3	.4	.6
Do Nothing	.8	.25	.3

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Basic Assumption

- **Actions with a higher expected value should accordingly have a higher probability of being performed**

if

$$E(\mathit{punishBully}, \mathit{StrictTeacher}) > E(\mathit{doNothing}, \mathit{StrictTeacher})$$

then

$$P(\mathit{punishBully} \mid \mathit{StrictTeacher}) > P(\mathit{doNothing} \mid \mathit{StrictTeacher})$$

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Method 1: Expected Value Ratio

- **Relative expected value is good overall indicator of probability**

$$P_{ratio}(PunishBully \mid StrictTeacher) = \frac{E(PunishBully, StrictTeacher)}{\sum_i E(action_i, StrictTeacher)}$$

Table of Expected Values

Action	Lax	Fair	Strict
Punish Bully	.5	.75	.75
Punish Class	.4	.6	.5
Punish Observer	.3	.4	.6
Do Nothing	.8	.25	.3

$$\frac{.75}{.75+.5+.6+.3} = .349$$

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Ranking-Based Methods

- Relative ranking or order is good overall indicator of probability
- Convert Expected Value to Ranking

Table of Expected Values

Action	Lax	Fair	Strict
Punish Bully	.5	.75	.75
Punish Class	.4	.6	.5
Punish Observer	.3	.4	.6
Do Nothing	.8	.25	.3

Table of Rankings

Action	Lax	Fair	Strict
Punish Bully	3	4	4
Punish Class	2	3	2
Punish Observer	1	2	3
Do Nothing	4	1	1

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Linear and Exponential Ranking Methods

$$P_{rank}(PunishBully \mid StrictTeacher) = \frac{Rank(PunishBully, StrictTeacher)}{\sum_i Rank(action_i, StrictTeacher)}$$

$$P_{\exp rank}(PunishBully \mid StrictTeacher) = \frac{e^{Rank(PunishBully, StrictTeacher)}}{\sum_i e^{Rank(action_i, StrictTeacher)}}$$

Table of Rankings

Action	Lax	Fair	Strict
Punish Bully	3	4	4
Punish Class	2	3	2
Punish Observer	1	2	3
Do Nothing	4	1	1

Linear

$$\frac{4}{1+2+3+4} = .4$$

Exponential

$$\frac{e^4}{e^1+e^2+e^3+e^4} = .644$$

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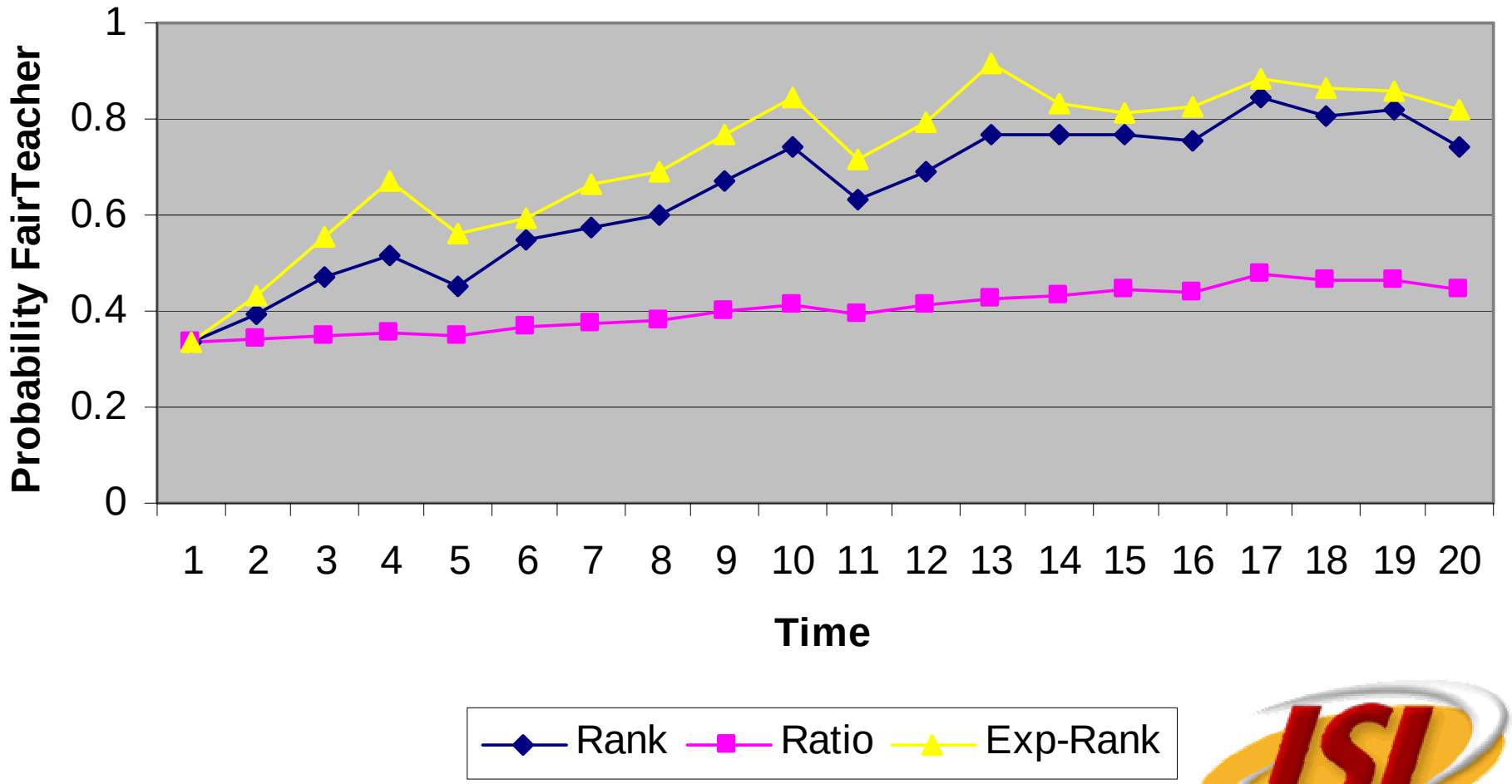
Fair Teacher



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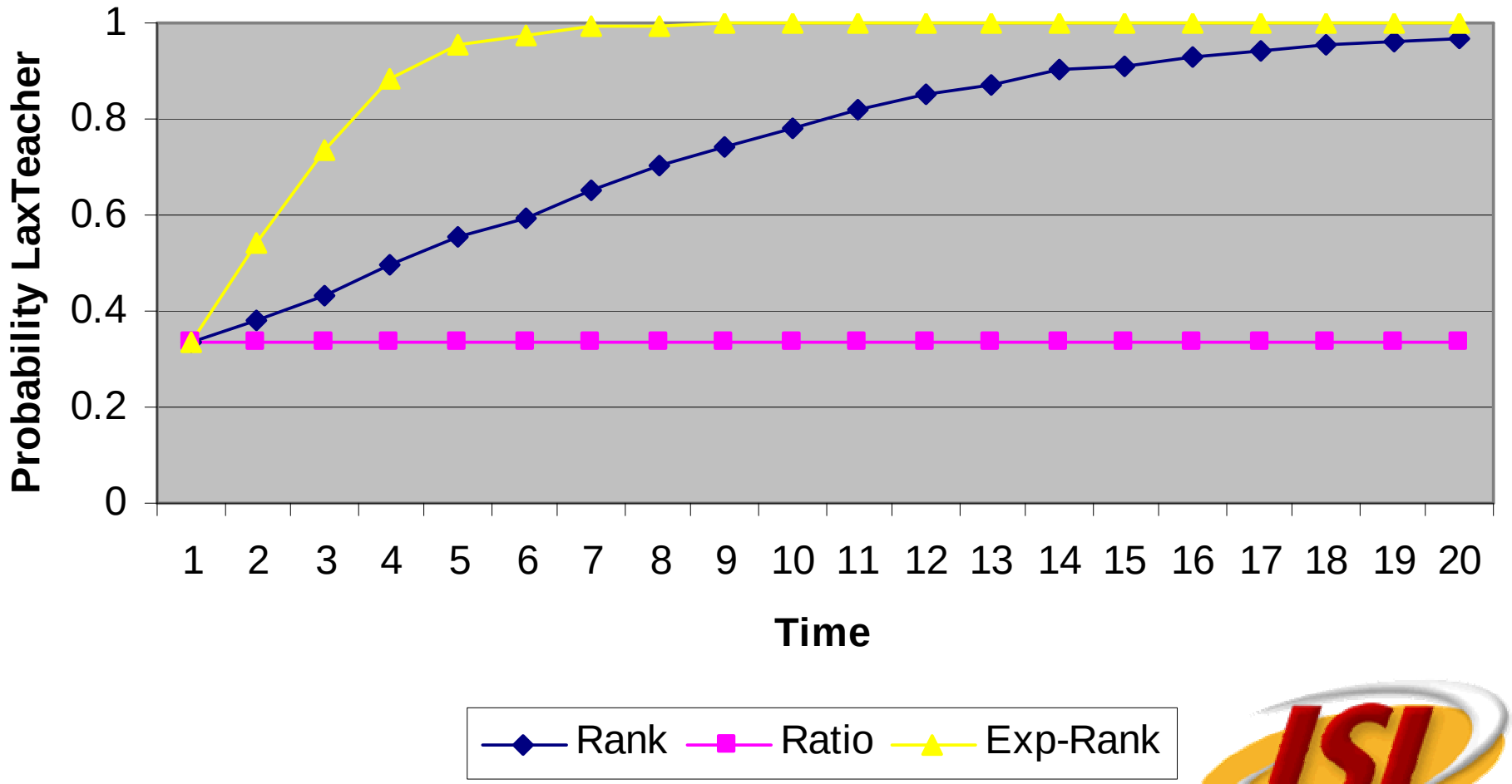
Lax Teacher



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No Convergence in Ratio Method

- No additional preference is given for optimal actions

Expected Value Table

Observed action →

Action	Lax	Strict	Fair
Do Nothing	.9	.7	.3
Punish Class	.8	.2	.6
Punish Bully	.6	.8	.75
Punish Onlooker	.4	.4	.4

$$P_{ratio}(Nothing \mid Lax) = .33 \qquad P_{ratio}(Nothing \mid Strict) = .33$$

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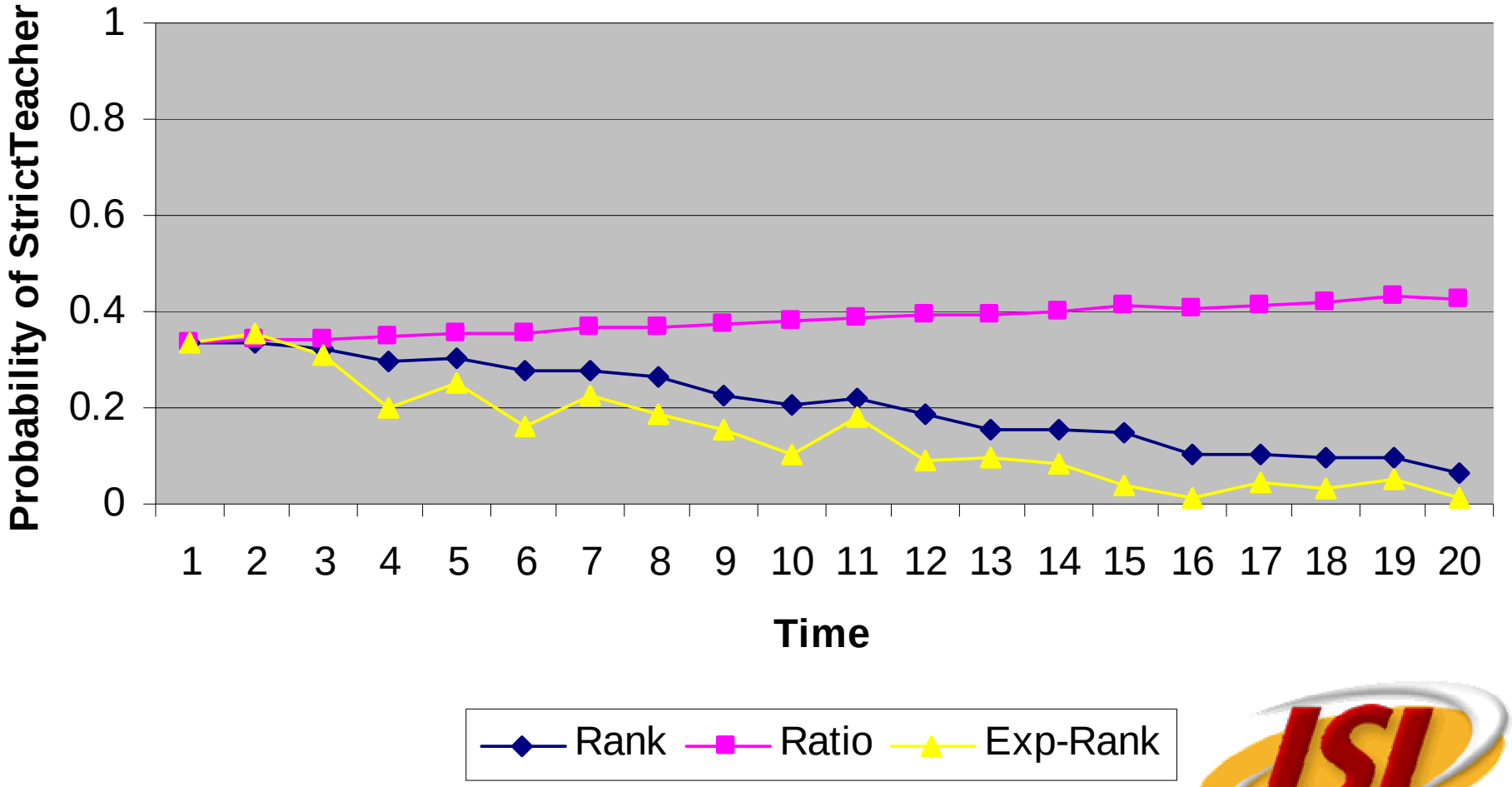
Strict Teacher



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What's Wrong with Ranking Methods?

- No notion of closeness

Expected Value Table

Action	Lax	Strict	Fair
Do Nothing	.9	.86	.3
Punish Class	.8	.89	.9
Punish Bully	.6	.88	.7
Punish Onlooker	.4	.87	.65

Ranking Table

Action	Lax	Strict	Fair
Do Nothing	4	1	1
Punish Class	3	4	4
Punish Bully	2	3	3
Punish Onlooker	1	2	2

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Discussion of Results

- **Ratio method**
  - Relative EV of action is accurate predictor of probability
  - Can converge slowly if EVs of actions are similar within model – no extra weight given to optimal actions
- **Ranking methods**
  - Relative ordering of actions is accurate predictor of probability
  - Much quicker convergence
  - Loses the notion of 'closeness'
- **Possible solution: Normalization across models!**

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Summary

- **Importance of mental models in constraining space**
- **Maintaining posterior probabilities over mental models**
  - Methods of calculating conditional probabilities:
    - Expected Value Ratio
    - Linear and Exponential Ranking methods
- **Preliminary experiments**
- **Identified boundary cases and issues with current methods of conditional probability calculation**

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Future Directions

- Better methods of calculating conditional probability that deal with issues of 'closeness' and of preference of optimal actions
- More formal characterization of conditional probability calculation methods
- Imperfect memory of observations

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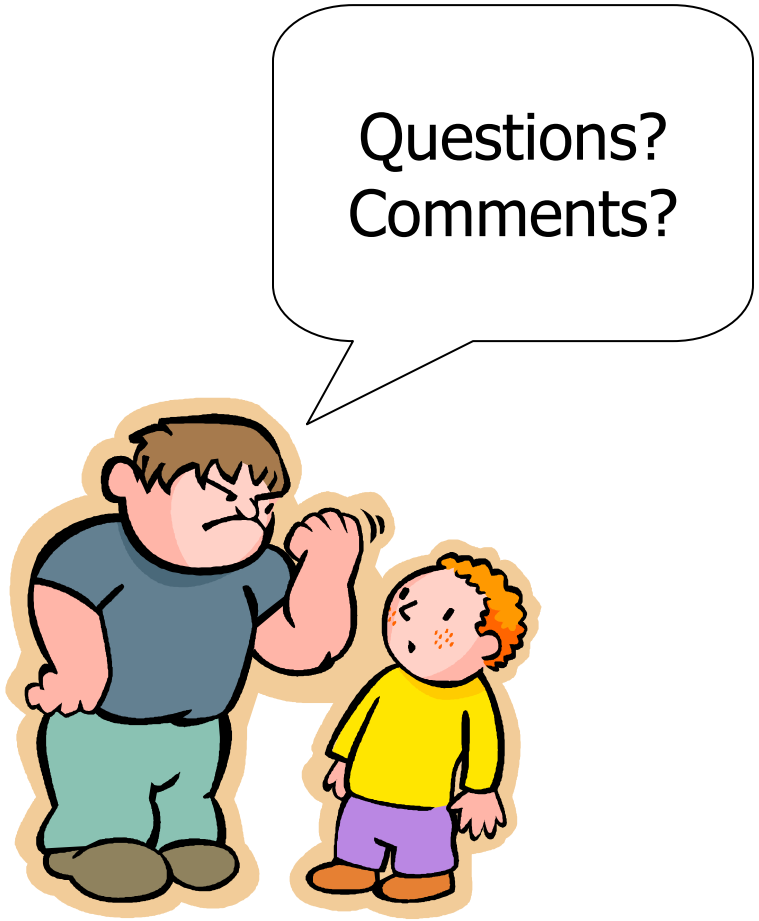

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